An Effective State-Space Feedback Autopilot for Ship Motion Control

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Abstract— **Ship autopilots are usually designed based on PID controller because of the simplicity and the ease of construct. However its performances in various environmental conditions are not as good as desired. To improve the ship performances, a linear state space feedback autopilot is designed. The paper presents the utility of this autopilot to stabilize the ship, shaping its yaw response and force the system to follow a desired path. The state space model of the 4DOF ship motion is presented and the method of the autopilot designing is completely described. Furthermore numerical simulations using real ship parameters are provided to illustrate the effectiveness of the proposed methodology. For the ship path following, a comparison between PID and feedback gain controllers is given to show the robustness of each controller in presence of wave disturbances.**

Keywords- Ship Control; Yaw Shaping; State-space Model; Nomoto Model; Linear Control; Ship Path Following; Disturbance Rejection

I. INTRODUCTION

Marine vehicles are designed to operate with adequate reliability and economy. And in order to accomplish this, it is essential to control the motion of the ship. Despite the external disturbances (waves, wind, currents), the control task consists in making the ship to follow, as closely as possible, a desired trajectory, which can be defined in terms of the ship's position, velocity and acceleration [1].

Over the last two decades, the ship control had received considerable research efforts and had undergone a period of significant progress, especially for roll and yaw motions. This was the work of different groups of research which targeted the development in different countries as: The Netherlands, Denmark, Sweden, the USA, Japan and China.

Conventional ships which are moving with constant forward speed, the steering machine is the only system to control the ship's motion. The principal function of the steering machine is to move the rudder to a desired angle when demanded by the autopilot system or by the helmsman. The main purpose (or the helmsman) of the autopilot is to control the heading of the ship affected by disturbances [2].

In actual ship building field, a considerable number of the vessels traveling oceans of the world are only equipped with rudders [3]. In this paper, it is assumed that the ship is moving at a constant forward speed, and only the rudder is controllable. The ship control problem is how to make the ship keep the desired trajectory by controlling the rudder's actuator.

To design an autopilot for the ship's motion is always a challenging problem. The motion is influenced by unpredictable environmental disturbances such as waves, winds, currents, change of depth under keel, etc. as well as ship sailing conditions such as speed, loading condition, trim, etc. To design an autopilot capable of taking into account all these factors is nowadays possible by different control design methods [4]. The autopilot takes into account the yaw-angle variations and it generates rudder commands to reject the disturbances according to the heading control law.

Many control methods are used to improve the ship performances especially against the disturbances, but the implementation of these methods are still difficult for conventional systems [5, 6]. However, state-space feedback gain, as one of the simplest methods, requires only a linear state-space model of the plant to be applied and implemented for a controllable system.

Because the necessity of the system model, existing mathematical ship models were developed to incorporate the use of all kind of controllers that could be commanded in a strategic manner. The models are still complex to use in control; many studies tried some modifications and simplifications to make it useful and easy to be controlled. Adopting these models, some theoretical study results have been achieved on the ship control motion [6].

In this work, a linear model for underactuated ship motion is presented in horizontal plane (4 Degree Of Freedom). This model is developed to state space representation that allows applying the feedback controller to shape ship motion as desired. To design the feedback controller, we will focus on the pole placement method. The procedure consists of two main steps: (i) proving the controllability of the system then writing its canonical form; (ii) finding the state-feedback gain by fixing the polynomial of the desired response. The principal role of the controller is to improve the ship performances in terms of effectiveness and robustness whatever the sailing conditions. The simplicity of the controller, comparatively with the modern and intelligent theory of control, can be deduced during the step of the feedback controller design.

The rest of the paper is organized as follows. System of interest and problem formulation is presented in the next section. Section 3 devotes to state-space modeling and section 4 describes the feedback control. Design procedure is described in section 5 while results and discussion are given in Section 6. Section 7 concludes the paper.

II. SYSTEM OF INTEREST AND PROBLEM FORMULATION

It is known that to fully represent a rigid body motion in space requires a Six Degree Of Freedom (6DOF) approach. This is the same case for the ship as a rigid body moving in sea.

Fig. 1 represents the Standard Notation followed by the Society of Naval Architects and Marine Engineers notation convention (SNAME. 1950) applied on the ship motion (6DOF).

This notation employs a combination of two separate right-handed orthogonal coordinate systems: one attached to the

earth and a second attached to the body of the ship. The TABLE I describe the different parameters of the Fig. 1.

The equations of motion describing the dynamics of a ship are readily obtained from Newton's law [5]:

$$\begin{cases}\sum Forces = \dfrac{d(mV)}{dt} \\ \sum Moments = \dfrac{d(I\omega)}{dt}\end{cases} \quad (1)$$

Where: m: mass of the body. V: translational velocity. I: Inertial moment. ω: rotational velocity.

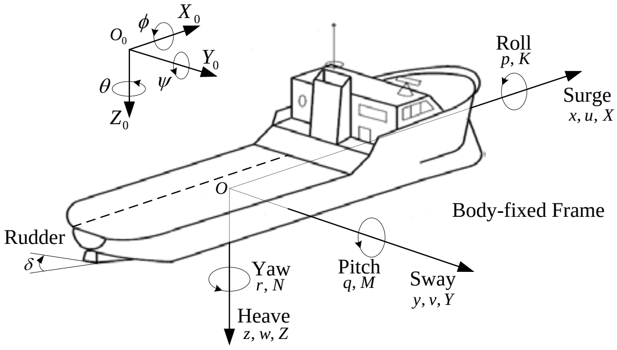

Figure 1 Standard notation and sign conventions for ship motion description (SNAME, 1950)

TABLE I COORDINATE FRAME VESSEL PARAMETERS

Parameters	Description
x, y, z	Distance along body-fixed system axes $[m]$
u_0, v, w	Translational velocity in body-fixed frame $[m/s]$
Xf, Yf, Zf	Force components along body axes $[kg]$
p, q, r	Rotational velocity in body-fixed frame $[rad/s]$
K, M, N	Moment components along body axes $[kg.m]$
ψ	Yaw angle $[rad]$
θ	Pitch angle $[rad]$
ϕ	Roll angle $[rad]$
δ	Rudder angle $[rad]$

Applying (1) on the ship, the motion in earth-fixed frame is described as followed:

$$\begin{cases} f_x(u,\dot{u},v,\dot{v},w,\dot{w},p,\dot{p},q,\dot{q},r,\dot{r},\delta) = Xf \\ f_y(u,\dot{u},v,\dot{v},w,\dot{w},p,\dot{p},q,\dot{q},r,\dot{r},\delta) = Yf \\ f_z(u,\dot{u},v,\dot{v},w,\dot{w},p,\dot{p},q,\dot{q},r,\dot{r},\delta) = Zf \\ f_\phi(u,\dot{u},v,\dot{v},w,\dot{w},p,\dot{p},q,\dot{q},r,\dot{r},\delta) = K \\ f_\theta(u,\dot{u},v,\dot{v},w,\dot{w},p,\dot{p},q,\dot{q},r,\dot{r},\delta) = M \\ f_\psi(u,\dot{u},v,\dot{v},w,\dot{w},p,\dot{p},q,\dot{q},r,\dot{r},\delta) = N \end{cases} \quad (2)$$

However a 3DOF plane motion is usually considered adequate for ship maneuvering study (surge, sway and yaw) [7], and for high speed vessels, roll mode is not negligible. Hence a 4DOF description that includes surge, sway, yaw and roll modes is needed [8], (Fig. 2).

Upon linearization with respect to straight line motion with forward speed u_0, the surge equation is decoupled and the following linear coupled sway-yaw-roll equations follow immediately [8].

For simplicity we assume that the only external force and moment are caused by a single rudder angle noted by δ, where:

$$\begin{bmatrix} Yf & N & 0 & K & 0 \end{bmatrix}^T = \begin{bmatrix} Y_\delta & N_\delta & 0 & K_\delta & 0 \end{bmatrix}^T \quad (3)$$

Referring to Fig. 2, we have:

$$\begin{cases} m(\dot{v}+u_0 r) = Y_{\dot{v}}\dot{v} + Y_v v + Y_{\dot{r}}\dot{r} + Y_r r + Y_{\dot{p}}\dot{p} + Y_p p + Y_\phi \phi + Y_\delta \delta \\ I_z \ddot{\psi} = N_{\dot{v}}\dot{v} + N_v v + N_{\dot{r}}\dot{r} + N_r r + N_{\dot{p}}\dot{p} + N_p p + N_\phi \phi + N_\delta \delta \\ \dot{\psi} = r \\ I_x \ddot{\phi} = K_{\dot{v}}\dot{v} + K_v v + K_{\dot{r}}\dot{r} + K_r r + K_{\dot{p}}\dot{p} + K_p p - mg\,\overline{GM}\,\phi + K_\delta \delta \\ \dot{\phi} = p \end{cases} \quad (4)$$

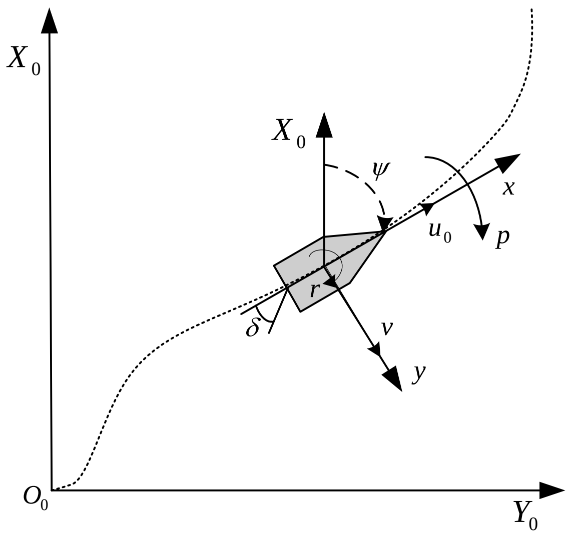

Figure 2 surge-sway-yaw -roll motion coordinate system

Where:

$Y_{\dot{v}}, N_{\dot{v}}, K_{\dot{v}}$..... : indicate the hydrodynamic coefficients. For instance, $Y_{\dot{v}}$ indicates the derivative of sway force Y to the sway acceleration $\dot{v}$ evaluated at the reference condition.

m: is the mass of the ship $[kg.s^2/m]$.

I_x, I_z: are the moments of inertia about x-axis and z-axis respectively $[kg.s^2.m]$.

g: Acceleration of gravity $[m/s^2]$.

$\overline{GM}$: is the metacentric height $[m]$, which indicates the restoring capability of a ship in rolling motion [9].

From (4) we rewrite:

$$\begin{cases} \dot{v} = Y'_{\dot{v}}\dot{v} + Y'_v v + Y_{\dot{r}}\dot{r} + Y'_r r + Y'_{\dot{p}}\dot{p} + Y'_p p + Y'_\phi \phi + Y'_\delta \delta \\ \dot{r} = N'_{\dot{v}}\dot{v} + N'_v v + N'_{\dot{r}}\dot{r} + N_r r + N'_{\dot{p}}\dot{p} + N'_p p + N'_\phi \phi + N'_\delta \delta \\ \dot{\psi} = r \\ \dot{p} = K'_{\dot{v}}\dot{v} + K'_v v + K'_{\dot{r}}\dot{r} + K_r r + K'_{\dot{p}}\dot{p} + K'_p p + K'_\phi \phi + K'_\delta \delta \\ \dot{\phi} = p \end{cases} \quad (5)$$

The equations in (5) present the linear model of the coupled steering and roll dynamics.

III. STATE SPACE MODELING

State space representation is a mathematical model of a physical system as a set of input, output and state variables related by first-order differential equations. The state space representation provides a convenient and compact way to model and analyze systems with multiple inputs and outputs.

For notational convenience, we define the state vector as:

$$x = \begin{bmatrix} v & r & \psi & p & \phi \end{bmatrix}^T \tag{6}$$

Equation (5) can be written in matrix form as followed:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} Y'_v & Y'_r & 0 & Y'_p & Y'_\phi \\ N'_v & N'_r & 0 & N'_p & N'_\phi \\ 0 & 1 & 0 & 0 & 0 \\ K'_v & K'_r & 0 & K'_p & K'_\phi \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} 0 & Y'_{\dot{r}} & 0 & Y'_{\dot{v}} & 0 \\ N'_{\dot{v}} & 0 & 0 & N'_{\dot{v}} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ K'_{\dot{v}} & K'_r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} Y'_\delta \\ N'_\delta \\ 0 \\ K'_\delta \\ 0 \end{bmatrix} \delta \tag{7}$$

Hence:

$$\underbrace{\begin{bmatrix} 1 & -Y'_{\dot{r}} & 0 & -Y'_{\dot{v}} & 0 \\ -N'_{\dot{v}} & 1 & 0 & -N'_{\dot{v}} & 0 \\ 0 & 0 & 1 & 0 & 0 \\ -K'_{\dot{v}} & -K'_r & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}}_{M} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} Y'_v & Y'_r & 0 & Y'_p & Y'_\phi \\ N'_v & N'_r & 0 & N'_p & N'_\phi \\ 0 & 1 & 0 & 0 & 0 \\ K'_v & K'_r & 0 & K'_p & K'_\phi \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} Y'_\delta \\ N'_\delta \\ 0 \\ K'_\delta \\ 0 \end{bmatrix} \delta \tag{8}$$

If M^{-1} exists, we have:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \left[\begin{array}{ccc|cc} a_{11} & a_{12} & 0 & a_{14} & a_{15} \\ a_{21} & a_{22} & 0 & a_{24} & a_{25} \\ 0 & 1 & 0 & 0 & 0 \\ \hline a_{41} & a_{42} & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & 1 & 0 \end{array}\right] \begin{bmatrix} v \\ r \\ \psi \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} b_1 \\ b_3 \\ 0 \\ b_4 \\ 0 \end{bmatrix} \delta \tag{9}$$

To simplify the system for further analysis, we can reorganize the state vector again such that state variables associated with the yaw and roll dynamics are separated. Moreover, (9) can be rewritten as:

$$\begin{bmatrix} \dot{x}_\psi \\ \dot{x}_\phi \end{bmatrix} = \begin{bmatrix} A_{\psi\psi} & A_{\psi\phi} \\ A_{\phi\psi} & A_{\phi\phi} \end{bmatrix} \begin{bmatrix} x_\psi \\ x_\phi \end{bmatrix} + \begin{bmatrix} b_\psi \\ b_\phi \end{bmatrix} \delta \tag{10}$$

Separating the dynamics, we can write:

$$\begin{cases} \dot{x}_\psi = A_{\psi\psi} x_\psi + A_{\psi\phi} x_\phi + b_\psi \delta & \text{Sway - Yaw dynamics} \\ \dot{x}_\phi = A_{\phi\psi} x_\psi + A_{\phi\phi} x_\phi + b_\phi \delta & \text{Roll Dynamics} \end{cases} \tag{11}$$

Neglecting the coupling matrices [2] $(A_{\psi\phi} = 0,\ A_{\phi\psi} = 0)$ implies that:

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix} \delta \tag{12}$$

Equation (12) is recognized as the Nomoto model. This model is widely employed in the ship steering autopilot design. Its simplicity and reasonable accuracy in describing small rudder angle yaw dynamics make them attractive [9].

If we consider the yaw angle as the system output $(\dot{\psi} = r)$, the full system will be presented in state space model as:

$$\begin{cases} \begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \\ \dot{p} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & a_{45} \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \\ p \\ \phi \end{bmatrix} + \begin{bmatrix} b_1 \\ b_3 \\ 0 \\ b_4 \\ 0 \end{bmatrix} \delta \\ y = \psi \end{cases} \tag{13}$$

Linearization of the ship nonlinear model about u_0 (service speed) implies that we can write the linearized model in standard state space form:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{14}$$

One of the important results that we can extract from the state space model is the stability of the system. The system is stable if and only if the eigenvalues of the matrix A have a negative real part.

We can extract other results as: the controllability, observability and identifiability. All of these characteristics can be demonstrated for the system.

IV. FEEDBACK CONTROL

For the controllable system, the design of state feedback control yields desirable closed-loop performance in terms of both transient and steady-state response characteristics. If the system is not, the state feedback can be used to stabilize the closed-loop state equation [10].

The state feedback control law features a constant state feedback gain matrix K and a new external reference input $r(t)$ (Fig. 3).

Equation (14) represents the open-loop system to be controlled. So, the focus is on the application of state feedback control laws of the form:

$$u(t) = -Kx(t) + r(t) \tag{15}$$

With the goal of achieving desired performance characteristics for the closed-loop state equation:

$$\begin{cases} \dot{x}(t) = (A - BK)x(t) + Br(t) \\ y(t) = Cx(t) \end{cases} \tag{16}$$

A fourth order linear state space model representing the sway-yaw-roll modes of motion is given in Ref. [11]; we can add the equation characterizing the yaw angle $(\dot{\psi} = r)$ and the full model will be:

$$A = \begin{bmatrix} -0.04 & -1.933 & 0 & 0 & -0.0756 \\ -0.00011 & -0.0813 & 0 & 0 & 0.001134 \\ 0 & 1 & 0 & 0 & 0 \\ -0.00594 & 0 & 0 & -0.07 & -0.059 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}; B = \begin{bmatrix} 0.1559 \\ -0.0033 \\ 0 \\ 0.00821 \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

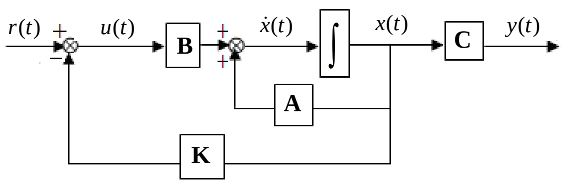

Figure 3 Basic structure of a state-feedback control system

V. DESIGN PROCEDURE

A. *Step 1: Controllability*

The controllability is an important property of a control system, and it plays a crucial role in many control problems, such as stabilization of unstable systems by feedback, or optimal control.

Roughly speaking, the concept of controllability denotes the ability to move a system around in its entire configuration space using only certain admissible manipulations [12].

The first step of designing is to prove that the pair (A, B) is controllable. If it is, the vector K exists.

The controllability matrix is defined as:

$$P = \begin{bmatrix} B & AB & A^2B & A^3B & A^4B \end{bmatrix} \quad (17)$$

The system is controllable if the matrix P has a full-row rank (it means: $rank(P) = order(A) = 5$).

We have:

$$P = \begin{bmatrix} 0.1559 & 0 & -0.0129 & 0.0003 & 0.0017 \\ -0.0033 & 0.0003 & 0.0002 & 0 & 0 \\ 0 & -0.0033 & 0.0003 & 0.0002 & 0 \\ 0.0082 & -0.0015 & -0.0096 & 0.0008 & 0.0013 \\ 0 & 0.1641 & -0.0015 & -0.0225 & 0.0011 \end{bmatrix}$$

$rank(P) = 5$, hence the system is controllable and the vector K exists.

B. *Step 2: Canonical form*

The second step is to find the matrix T that transforms the system in its canonical form.

The system will be represented as:

$$\begin{cases} \dot{z}(t) = (A_c - B_c K_c) z(t) + B_c r(t) \\ y(t) = C_c z(t) \end{cases} \quad (18)$$

Where: $z(t) = Tx(t)$

This gives:

$$A_c = TAT^{-1}, \; B_c = TB, \; C_c = CT^{-1} \text{ and } K_c = KT^{-1}$$

Note that all the followed results are computed by Matlab software.

For the considered system, it is found that the matrix of transformation to the canonical form is:

$$T = \begin{bmatrix} -5.08 & -553.41 & -54.82 & -4.12 & -0.29 \\ -30.83 & -14247.78 & -1578.93 & 127.67 & -2.41 \\ -3159.15 & -145938.58 & -11227.33 & 1329.31 & 12.48 \\ -3159.15 & -146365.84 & -14697.75 & 1329.31 & -56.37 \\ -22155.03 & -1021468.36 & -76830.91 & 10133.67 & 130.18 \end{bmatrix}$$

And we have:

$$A_C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -0.0007 \\ 0 & 1 & 0 & 0 & -0.0206 \\ 0 & 0 & 1 & 0 & -0.1461 \\ 0 & 0 & 0 & 1 & -0.1913 \end{bmatrix} \quad B_C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$C_C = \begin{bmatrix} -0.0033 & 0.0002 & 0.0002 & 0 & 0 \end{bmatrix}$$

In next step, we adopt the canonical form of the system and our goal will be to find K_c instead of K.

C. *Step 3: feedback vector*

In this step, the feedback vector K_c is expressed in terms of the open-loop and closed-loop desired dynamic parameters.

The open-loop dynamics is characterized by:

$$|sI - A_c| = s^5 + 0.19s^4 + 0.14s^3 + 0.026s^2 + 0.0007s \quad (19)$$

We denote the feedback gain K_c as:

$$K_c = \begin{bmatrix} \delta_0 & \delta_1 & \delta_2 & \delta_3 & \delta_4 \end{bmatrix} \quad (20)$$

As result, the closed-loop dynamics is characterized by:

$$\begin{aligned} |sI - (A_c - B_c K_c)| &= s^5 + (\delta_4 + 0.19)s^4 + (\delta_3 + 0.14)s^3 \\ &+ (\delta_2 + 0.026)s^2 + (\delta_1 + 0.007)s + \delta_0 \end{aligned} \quad (21)$$

The desired dynamics is expressed by a five-order polynomial as followed:

$$\alpha(s) = s^5 + \alpha_4 s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s^1 + \alpha_0 \quad (22)$$

Equalizing the desired dynamics polynomial (22) with closed-loop dynamics polynomial (21), we can write the vector K_c as:

$$\begin{aligned} K_c = [\alpha_0 \quad & (\alpha_1 - 0.0007) \quad (\alpha_2 - 0.0206) \\ & (\alpha_3 - 0.1461) \quad (\alpha_4 - 0.1913)] \end{aligned} \quad (23)$$

Now, it only remains to find the coefficients α_i.

D. *Step 4: Dominant poles approach*

The mathematical formulation of the parameters as: rise time t_R, settling time t_s, percent overshoot PO etc., is not known for high order system ($order > 2$). Hence, many studies tried to approximate the high order system to first or second order system if possible.

The five order polynomial in (22) can be rewritten as:

$$\alpha(s) = (s - \underline{\underline{P_0}})(s - \underline{\underline{P_1}})(s - P_2)(s - P_3)(s - P_4) \quad (24)$$

In this work, the dynamic of yaw ship motion is shaped as second order system dynamics (it can be shaped also as first order system response). Second order system response is usually characterized by two parameters noted ξ and ω_n.

We consider only P_0 and P_1 of (24) and we neglect P_2, P_3 and P_4.

We write:

$$\begin{cases} P_0 = -\xi\omega_n - \omega_n j\sqrt{1-\xi^2} \\ P_1 = -\xi\omega_n + \omega_n j\sqrt{1-\xi^2} \\ P_2 = P_3 = P_4 = 10 * \mathrm{Re}(P_0) \end{cases} \quad (25)$$

Note that:

$j = \sqrt{-1}$, Re : real part of P_0

ξ : dimensionless damping ratio.

$\omega_n [rad.s^{-1}]$: fixed undamped natural frequency.

For dominant poles approach, we take P_2 , P_3 and P_4 ten times P_0 (or more) to neglect their dynamics and the desired behavior will obey only to two poles: P_0 and P_1 .

E. *Step 5: Shpaing the dyanmics*

The last step is to specify desired closed-loop system behavior via (P_0, P_1) selection; it is called: shaping the dynamic response. This choice fixes directly the values of feedback gain vector K .

It consists to fix the couple (ξ, ω_n) which characterizes the desired response. By fixing values of ξ and ω_n , it leads to fix the closed-loop characteristics of the transient response such as: rise time t_R , peak time t_P , percent overshoot PO , and settling time t_S .

For the ship yaw motion, we have taken the followed values:

$$\begin{cases} \xi = 0.8 \\ \omega_n = 0.02 \end{cases} \quad (26)$$

Where:

$$\begin{cases} t_R \cong \dfrac{2.16\xi + 0.60}{\omega_n} = 116.4 \\ t_S \cong \dfrac{4}{\xi\omega_n} = 250 \\ PO = 100 e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}} \cong 2\% \end{cases} \quad (27)$$

Hence:

$$\begin{cases} P_0 = -0.016 - 0.012 j \\ P_1 = -0.016 + 0.012 j \\ P_2 = P_3 = P_4 = -0.16 (or < -0.16) \end{cases} \quad (28)$$

The desired polynomial will be:

$$\alpha(s) = (s + 0.016 + 0.012 j)(s + 0.016 - 0.012 j)(s + 0.2)(s + 0.3)(s + 0.4) \quad (29)$$

We rewrite (29) as:

$$\alpha(s) = s^5 + 0.932 s^4 + 0.2892 s^3 + 0.0327 s^2 + 0.0009 s + 0.0001 \quad (30)$$

From (23), we write the vector gain K_c as:

$$K_c = [0.00001 \;\; 0.0002 \;\; 0.0121 \;\; 0.1431 \;\; 0.7407]$$

Finally, the desired feedback vector is:

$$K = K_c T \quad (31)$$

VI. RESULTS OF SIMULATION

We perform some numerical simulations to illustrate the effectiveness and the robustness of the proposed methodology of control.

In this section, we present some results using the given model for the naval vessel and the environmental disturbances.

All the results are computed with the aid of MATLAB software.

A. *Directional stability*

If we look to the state matrix A of the considered system, the third row vector is null, this indicates that the system has a null eigenvalue. As result, the system is not stable.

If the yaw angle is changed by an external force, the open-loop system changes the direction and continues moving on this direction; however in close-loop case the system returns to the initial direction. We can see the influence of the feedback on the trajectory in the Fig. 4.

For the reader's convenience, the mathematical trajectory of the underactuated ship moving in surge, sway, and yaw (Fig. 4) is recaptured as:

$$\begin{cases} Xpos = \int (u_0 \cos(\psi) - v \sin(\psi)) dt \\ Ypos = \int (u_0 \sin(\psi) - v \cos(\psi)) dt \end{cases} \quad (32)$$

We can interpret the directional stability in the close-loop system by the asymptotic stability in yaw mode (Fig. 5). However the roll mode is stable in both cases, but the amplitude of damping increases in close-loop case (Fig. 6).

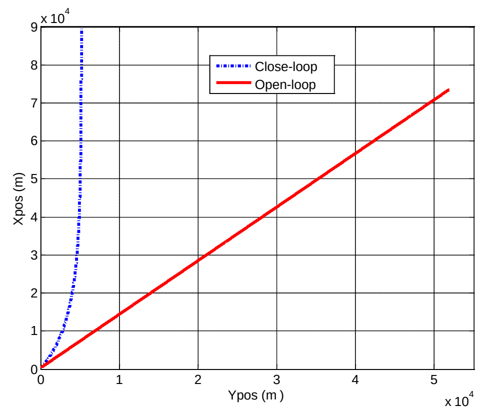

Figure 4 Directional stability of the trajectory

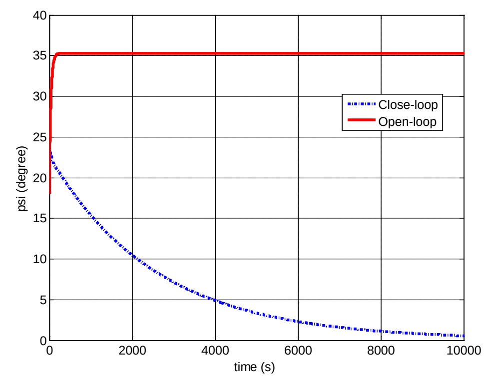

Figure 5 Yaw angle response in open and close-loop

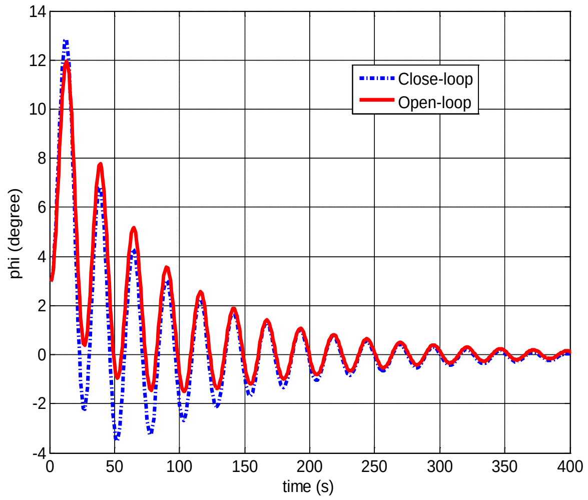

Figure 6 roll angle response in open and closed loop

Note that the initial conditions didn't present the equilibrium state $(x(0) \neq 0)$. For this reason, the yaw in open-loop case changes the direction, however for the close-loop case the ship return to equilibrium state (see the trajectory in Fig. 4).

For information, the initial state of the ship is taken as:

$$v = -1.5\,,\; r = 0.01\,,\; \psi = \frac{\pi}{10}\,,\; p = 0.001\,,\; \phi = \frac{\pi}{60} \quad \text{with} \;\; u_0 = 5$$

B. Shaping yaw dynamics

In addition to closed-loop asymptotic stability, we are often interested in other characteristics of the closed-loop transient response (t_R, t_P, PO, and t_S).

For instance, if we specify that the desired yaw response by PID controller and state-space feedback controller should have the smallest time constant with 2% overshoot, we get the results of Fig. 5.

Note that the yaw reference is taken: $\psi_d(t) = 100^0$.

The initial state of the ship is:

$$v = 0\,,\; r = 0\,,\; \psi = 0\,,\; p = 0\,,\; \phi = 0 \quad \text{with}\; u_0 = 5\,m.s^{-1}$$

It is clear in the yaw response (Fig. 7) that we can reduce the rise time from $t_R \cong 145s$ to $t_R \cong 110s$ by implementing a linear feedback controller instead of classical PID controller.

The yaw response in the case PID controller presents also the criteria of minimal rise time t_R with overshoot $PO = 2\%$ (Fig. 7 includes details).

The parameters of the PID controller are:

$$K_P = -0.23,\, K_I = 0.001,\, K_D = 0.09$$

However the amplitude of the roll damping increases for the feedback gain controller, which requires having a good equipment and efficient law to control roll motion (usually use fins to control the roll mode).

Remember that the pair (A, B) is controllable, which is a necessary condition to apply the state-space feedback controller.

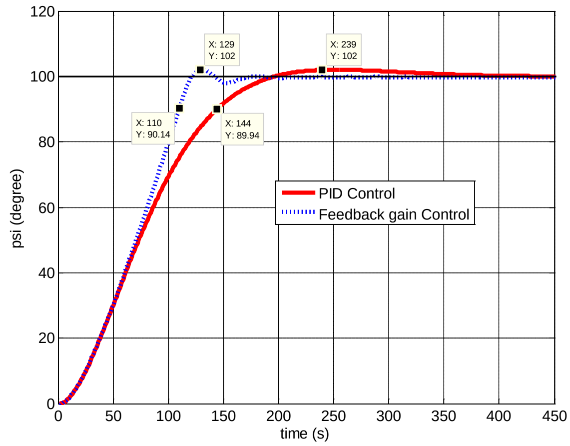

Figure 7 Shaping yaw response

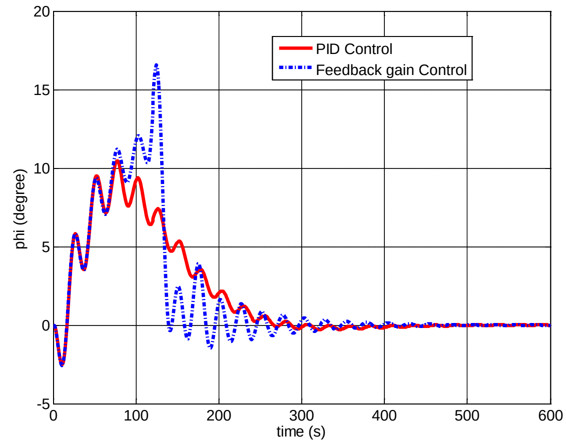

Figure 8 Control influence on the roll mode

C. Ship path following

In this section, simulation results will be presented to prove the performances of the proposed feedback autopilot. For this purpose, we try to force the ship to follow a desired path (zigzag path) and try to apply wave disturbances on the motion.

The results of simulation will prove the robustness of the two controllers in presence of disturbances.

Briefly, the approach is to steer the ship such that it eliminates the distance between itself and the desired path (Fig. 9).

We define the following variables to mathematically formulate the control objective:

$$\begin{cases} x_e = x_d - x \\ y_e = y_d - y \\ \psi_e = \psi_d - \psi \end{cases} \tag{33}$$

Where:

$$\psi_d = \arcsin\left(\frac{y_e}{\sqrt{x_e^2 + y_e^2}}\right) \tag{34}$$

The undesirable motion of a ship in a seaway is induced by the action of environmental disturbances: waves, wind and current. However, ocean waves are the dominant

environmental disturbances. As a result, the wave disturbances are considered at the output as it is shown in Fig. 10.

Waves' disturbances can be characterized in terms of their frequency spectrum. This frequency spectrum can be simulated by either a series of added sinusoidal signals, or using filtered white noise.

In our work, the used filter to approximate the spectrum is of the form Ref. [11]:

$$F(s) = \frac{s}{s^2 + 0.1s + 0.25} \tag{35}$$

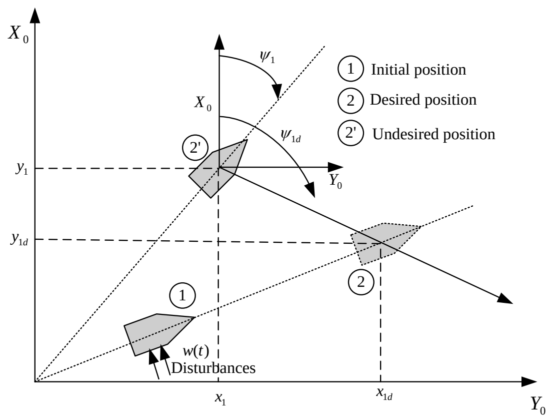

Figure 9 Deflection of the ship

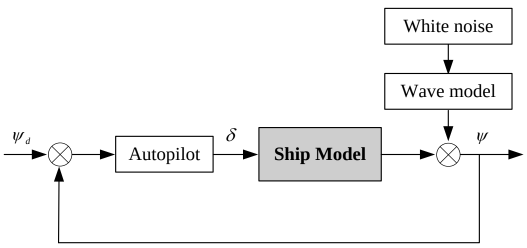

Figure 10 System structure with disturbances

Figs. 11 and 12 show $X-Y$ plots of the ship zigzag trajectory; the test is done for the two different autopilots.

For the test of tracking, we generate a zigzag path as desired trajectory. This test is a standard maneuver used to compare the maneuvering properties and control characteristics of a ship with those of other ships. Another feature is that the experimental results of the test can be used to calculate the parameters of Nomoto model.

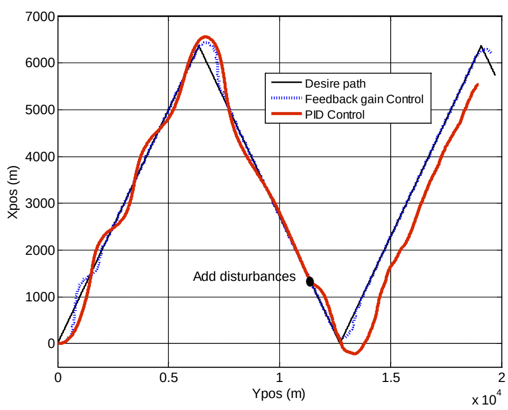

Figure 11 Zigzag path following with and without low waves disturbance

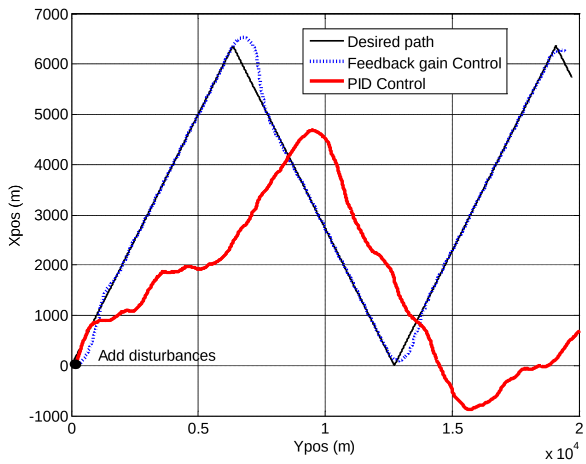

Figure 12 Zigzag path following with high waves disturbance

The results of two simulation tests are given here. The first simulation demonstrates the effectiveness of the feedback controller in absence of disturbance and its robustness comparatively with the PID autopilot in presence of disturbances (Fig. 11). In the second simulation, the influence of strong waves on the path following is demonstrated (Fig. 12).

From the first simulation results (Fig. 11) and in absence of disturbances, it can be concluded that feedback controller is more effective and faster than PID controller especially in terms of following the desired path. After adding small disturbances (white noise driving the filter has a power spectral density $\sigma^2 = 0.5$), the feedback autopilot keep the following of the path, however the PID controller can only keep the path direction not the path (keep only the yaw direction).

If we increase the waves' power ($\sigma^2 = 1$) the influence on the ship was presented in Fig. 12. In this case, the robustness of the feedback gain controller is completely proved.

VII. CONCLUSIONS

The method for constructing ship autopilots based on linear feedback gain has been presented. The advantages of this method are to construct the controller easily and to have some malleability to shape the yaw dynamics as desired. Simulation results with real ship parameters show that by linear feedback controller we decrease the time constant comparatively with PID controller, which is an important characteristic especially for ship maneuvering.

Simulation results show that this feedback autopilot is effective and robust in comparison with conventional autopilot based on PID control especially in the presence environmental disturbances (sea waves).

The benefits of the proposed approach lie in the ability of such desired polynomial to better shape the yaw response as desired and to reject external disturbances.

The presented approach can be readily applied to other vehicles or extended to higher dimensional control and guidance problems if the system is controllable.

Among the problems of decreasing the yaw rise time, we mention the increasing of the roll damping amplitude; this problem is due to the roll coupling. And it can be settled by studying yaw response and roll controllers together.

Another limitation of the state-space feedback controller is the controllability of the system which is necessary.

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